Warm Up:

1. $$\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5} = \frac{0}{0}$$

$$\lim_{x\to 5}\frac{(\sqrt{x-1}-2)}{(x-5)}\cdot\frac{(\sqrt{x-1}+2)}{(\sqrt{x-1}+2)}$$

$$\lim_{x\to 5}\frac{x-1-4}{(x-5)(\sqrt{x+1}+2)}$$

$$\lim_{x\to 5}\frac{\cancel{x-5}}{(\cancel{x-5})(\sqrt{x-1}+2)}$$

$$\lim_{x\to 5}\frac{1}{\sqrt{x-1}+2}$$

$$\frac{1}{\sqrt{4}+2}$$

$$\frac{1}{4}$$

2. $$\lim_{x\to 9}\frac{\sqrt{x+72}-3\sqrt{x}}{x^2-81}\cdot\frac{(\sqrt{x+72}+3\sqrt{x})}{(\sqrt{x+72}+3\sqrt{x})}$$

$$\lim_{x\to 9}\frac{x+72-9x}{(x^2-81)(\sqrt{x+72}+3\sqrt{x})}$$

$$\lim_{x\to 9}\frac{-8x+72}{(x^2-81)(\sqrt{x+72}+3\sqrt{x})}$$

$$\lim_{x\to 9}\frac{-8\cancel{(x-9)}}{(x+9)\cancel{(x-9)}(\sqrt{x+72}+3\sqrt{x})}$$

$$\lim_{x\to 9}\frac{-8}{(x+9)(\sqrt{x+72}+3\sqrt{x})}$$

$$\frac{-8}{18\cdot(9+9)}$$

$$\frac{-8}{324} = \frac{-2}{81}$$

Infinite Limits

Infinite limit theorem:

$$\lim_{x \to a} \frac{f_{(x)}}{g_{(x)}} = \frac{any\ number}{0}\ then,\ answer = +or -\infty$$

Determining the sign of the answer:

1. Find the sign of the numerator by substitution.

2. Find how the denominator approaches zero (from neg or pos #'s) by choosing values from the correct side and substituting.

3. Apply usual division rules.

Ex 1: $\lim\limits_{x \to 5^+} \dfrac{x+2}{x-5} = \dfrac{7}{0}$

$\dfrac{(+)}{(+)}$

∞

Ex 2: $\lim\limits_{x \to 2^-} \dfrac{x}{x-2} = \dfrac{2}{0}$

$\dfrac{(+)}{(-)}$

$-\infty$

Ex 3: $\lim\limits_{x \to -1^+} \dfrac{-1}{x+1} = \dfrac{-1}{0}$

$\dfrac{(\ )}{(+)}$

$-\infty$

Ex 4: $\lim\limits_{x \to 2} \dfrac{4}{(x-2)^2} = \dfrac{4}{0}$

$\lim\limits_{x \to 2^-} \dfrac{4}{(x-2)^2}$

$\dfrac{(+)}{(+)}$

∞

$\lim\limits_{x \to 2^+} \dfrac{4}{(x-2)^2}$

$\dfrac{(+)}{(+)}$

∞

∞

Special Trig Limits

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1 \qquad \lim_{x\to 0}\left(\frac{1-\cos x}{x}\right)=0$$

ex1. $\lim_{\theta\to 0}\left(\frac{\sin 2\theta}{\theta}\right) = 2$

ex2. $\lim_{\alpha\to 0}\left(\frac{\cos\alpha - 1}{\alpha}\right)$

$\lim_{\alpha\to 0} \frac{-1(1-\cos\alpha)}{\alpha} = 0$

ex3. $\lim_{x\to 0}\left(\frac{\sec(x)-1}{x\sec(x)}\right)$

$\lim_{x\to 0} \frac{\cancel{\sec x}}{x\cancel{\sec x}} - \frac{1}{x\sec x}$

$\lim_{x\to 0} \frac{1}{x} - \frac{\cos x}{x}$

$\lim_{x\to 0} \frac{1-\cos x}{x}$

0

ex4.

$\lim_{\beta\to 0}\left(\frac{\tan(\beta)\cos(\beta)\sin^2(\beta)+\tan(\beta)\cos^3(\beta)}{\beta}\right)$

$\lim_{\beta\to 0} \frac{\tan\beta\cos\beta(\sin^2\beta+\cos^2\beta)}{\beta}$

$\lim_{\beta\to 0} \frac{\tan\beta\cos\beta(1)}{\beta}$

$\lim_{\beta\to 0} \frac{\frac{\sin\beta}{\cancel{\cos\beta}}\cdot\frac{\cancel{\cos\beta}}{1}}{\beta}$

$\lim_{\beta\to 0} \frac{\sin\beta}{\beta}$

1

Ex 5 $\lim_{x\to 4^+} \dfrac{-x^2+x}{x^2-2x-8}$ $\dfrac{-12}{0}$

$\lim_{x\to 4^+} \dfrac{-x^2+x}{(x-4)(x+2)}$

$\dfrac{(-)}{(+)(+)}$

$-\infty$

Ex 6 $\lim_{x\to 3} \dfrac{x+3}{x^2-13x+30} = \dfrac{6}{0}$

$\lim_{x\to 3^-} \dfrac{x+3}{(x-10)(x-3)}$

$\dfrac{(+)}{(-)(-)}$

∞

$\lim_{x\to 3^+} \dfrac{x+3}{(x-10)(x-3)}$

$\dfrac{(+)}{(-)(+)}$

$-\infty$

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